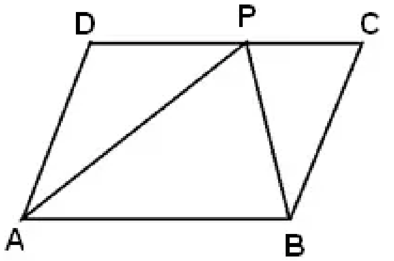
Chapter 21. Areas Theorems on Parallelograms

Ex 21.1

Answer 1.



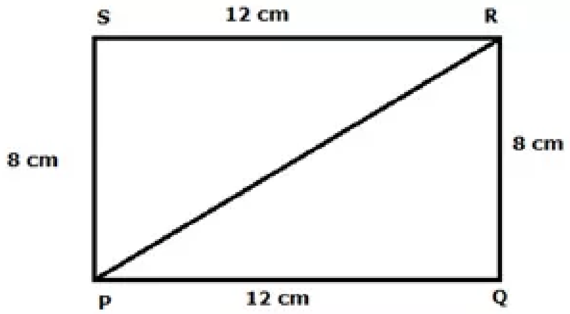
ar(Δ APB) = $\frac{1}{2}$ x ar(parallelogram ABCD)

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

ar(Δ APB) = $\frac{1}{2}$ x 60 cm^2

ar(Δ APB) = 30 cm^2

Answer 2.



Since PQRS is a rectangle, therefore PQ = SR.

SR = 12 cm

PS = 8 cm

ar(Δ PRS) = $\frac{1}{2}$ x base x height

ar(Δ PRS) = $\frac{1}{2}$ x SR x PS

ar(Δ PRS) = $\frac{1}{2}$ x 12 x 8

ar(Δ PRS) = 48 cm^2

Answer 3.

(i) ar(Δ QTS) = $\frac{1}{2}$ x ar(parallelogram QTSR)

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$\Rightarrow$ ar(parallelogram QTSR) = 2 x ar(Δ QTS)

$\Rightarrow$ar(parallelogram QTSR) = 2 x 60 cm^2

$\Rightarrow$ar(parallelogram QTSR) = 120 cm^2

(ii) ar(Δ QTS) = $\frac{1}{2}$ x ar(parallelogram QTSR)

ar(Δ QTS) = ar(ΔRSQ) = 60 cm^2

Now,

ar(Δ RSQ) = $\frac{1}{2}$ x ar(rectangle PQRS)

$\Rightarrow$ ar(rectangle PQRS) = 2 x ar(Δ RSQ)

$\Rightarrow$ar(rectangle PQRS) = 2 x 60 cm^2

$\Rightarrow$ar(rectangle PQRS) = 120 cm^2

(iii) Since PQRS is a rectangle,

Therefore RS = PQ .....(i)

QTSR is a parallelogram,

Therefore, RS = QT ......(ii)

From (i) and (ii)

PQ = QT ....(iii)

In Δ PSQ and Δ QST

QS = QS

PQ = QT (from (iii))

$\angle$PQS = $\angle$SQT = 90°

Therefore, Δ PSQ $\cong$ Δ QST

Area of two congruent triangles is equal.

Hence, ar(Δ PSQ) = ar(Δ QTS) = 60 cm^2

Answer 7.

$$ar(\Delta APD) = \frac{\sqrt{3}s^2}{4}$$

$$ar(\Delta APD) = \frac{\sqrt{3} \times 8^2}{4}$$

$$ar(\Delta APD) = \frac{\sqrt{3} \times 64}{4}$$

$$ar(\Delta APD) = \sqrt{3} \times 16 = 16\sqrt{3}cm^2$$

$$ar(\Delta APD) = \frac{1}{2} \text{ x } ar(\text{parallelogram } ABCD)$$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$\Rightarrow ar(\text{parallelogram } ABCD) = 2 \text{ x } ar(\Delta APD)$

$\Rightarrow ar(\text{parallelogram } ABCD) = 2 \text{ x } 16\sqrt{3} \text{ cm}^2$

$\Rightarrow ar(\text{parallelogram } ABCD) = 32\sqrt{3} \text{ cm}^2$

Answer 8.

(i) Area of a rectangle and area of a parallelogram on the same base is equal.

Here,

For rectangle PQMN, base = PQ

For parallelogram PQRS, base = PQ

Therefore, Area of rectangle PQMN = Area of parallelogram PQRS

Area of rectangle PQMN = 84 cm^2

(ii) $ar(\Delta PQS) = \frac{1}{2} \text{ x } ar(\text{parallelogram } PQRS)$

$ar(\Delta PQS) = \frac{1}{2} \text{ x } 84 \text{ cm}^2$

$ar(\Delta PQS) = 42 \text{ cm}^2$

(iii) $ar(\Delta PQN) = \frac{1}{2} \text{ x } ar(\text{rectangle } PQMN)$

$ar(\Delta PQN) = \frac{1}{2} \text{ x } 84 \text{ cm}^2$

$ar(\Delta PQN) = 42 \text{ cm}^2$

Answer 9.

In quadrilateral PQST,

$ar(\Delta PQS) = \frac{1}{2} \times ar(\text{quadrilateral } PQST)$

$ar(\text{quadrilateral } PQST) = 2ar(\Delta PQS)$ ....(i)

In ΔPSR,

$ar(\Delta PSR) = ar(\Delta PQS) + ar(\Delta QSR)$

but $ar(\Delta PQS) = ar(\Delta QSR)$ (since QS is median as QS||TP))

$ar(\Delta PSR) = 2ar(\Delta PQS)$ ....(ii)

From (i) and (ii)

$ar(\text{quadrilateral } PQST) = ar(\Delta PSR)$

Answer 14.

In parallelogram ABCD,

$ar(\Delta ABC) = \frac{1}{2} \times ar(\text{parallelogram } ABCD)$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$ar(\text{parallelogram } ABCD) = 2ar(\Delta ABC)$ ....(i)

In ΔACE,

$ar(\Delta ACE) = ar(\Delta ABC) + ar(\Delta BCE)$

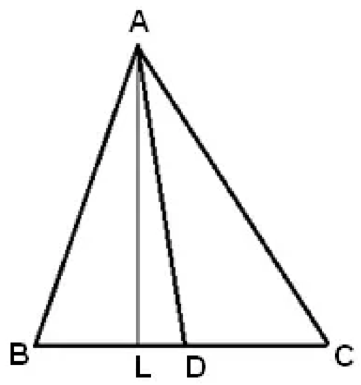
but $ar(\Delta ABC) = ar(\Delta BCE)$ (since BC is median)

$ar(\Delta ACE) = 2ar(\Delta ABC)$ ....(ii)

From (i) and (ii)

$ar(\text{parallelogram } ABCD) = ar(\Delta ACE)$

Answer 16.



Draw AL perpendicular to BC.

Since AD is median of ΔABC. Therefore, D is the mid-point of BC.

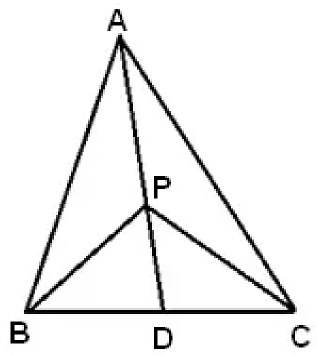
⇒ BD = DC

⇒BD x AL = DC x AL (multiplying by AL)

$\Rightarrow \frac{1}{2}(BD \times AL) = \frac{1}{2}(DC \times AL)$

⇒ar(Δ ABD) = ar(Δ ADC)

Answer 17.



AD is the median of Δ ABC. So, it will divide Δ ABC into two triangles of equal areas.

Therefore, Area (Δ ABD) = area (Δ ACD) ... (1)

Now PD is the median of ΔPBC.

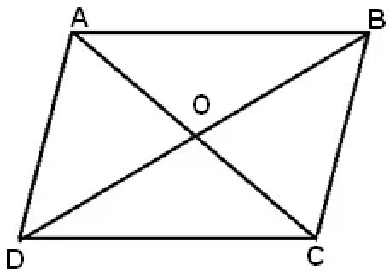
Therefore, Area (Δ PBD) = area (Δ PCD) ... (2)

Subtract equation (2) from equation (1), we have

Area (Δ ABD) – area (Δ PBD) = Area (Δ ACD) – Area (Δ PCD)

Area (ΔABP) = area (ΔACP)

Answer 19.



The diagonals of a parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

$\therefore$ ar(ΔAOB) = ar(ΔBOC).......(i)

In ΔBCD, CO is the median.

$\therefore$ ar(ΔBOC) = ar(ΔCOD).......(ii)

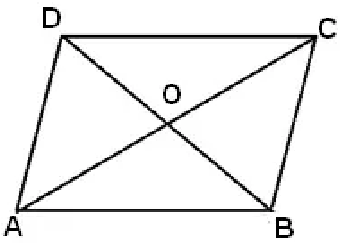
Similarly, ar(ΔCOD) = ar(ΔAOD)......(iii)

From (i), (ii) and (iii)

ar(ΔAOB) = ar(ΔBOC) = ar(ΔCOD) = ar(ΔAOD)

Hence, diagonals of a parallelogram divide it into four triangles of equal areas.

Answer 20.



In ΔABD,

BO = OD

$\Rightarrow$O is the mid-point of BD

$\Rightarrow$AO is a median.

$\Rightarrow$ar(ΔAOB) = ar(ΔAOD) ....(i)

In ΔCBD, O is the mid-point of BD

$\Rightarrow$CO is a median.

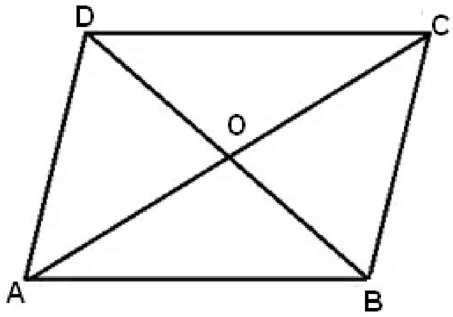
$\Rightarrow$ar(ΔCOB) = ar(ΔCOD) ....(ii)

Adding (i) and (ii)

ar(ΔAOB) + ar(ΔCOB) = ar(ΔAOD) + ar(ΔCOD)

Therefore, ar(ΔABC) = ar(ΔADC)

Answer 21.



Since the diagonals of a rhombus intersect at right angles,

Therefore, OB $\perp$ AC and OD $\perp$ AC

Now, ar(rhombus ABCD) = ar($\triangle$ABC) + ar($\triangle$ADC)

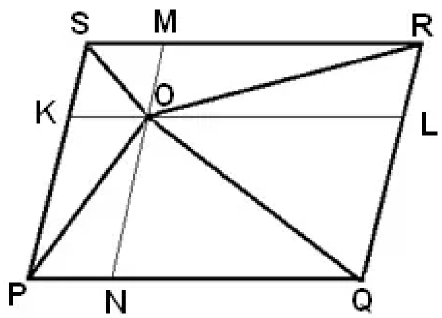
$$= \frac{1}{2}(AC \times BO) + \frac{1}{2}(AC \times DO)$$

$$= \frac{1}{2}\{AC \times (BO + DO)\}$$

$$= \frac{1}{2}(AC \times BD)$$

Therefore, the area of a rhombus is equal to half the rectangle contained by its diagonals.

Answer 22.



Let us draw a line segment KL, passing through point O and parallel to line segment PQ.

In parallelogram PQRS,

PQ || KL (By construction) ... (1)

PQRS is a parallelogram.

$\therefore$ PS || QR (Opposite sides of a parallelogram)

$\Rightarrow$ PK || QL ... (2)

From equations (1) and (2), we obtain

PQ || KL and PK || QL

Therefore, quadrilateral PQLK is a parallelogram.

It can be observed that ΔPOQ and parallelogram PQLK are lying on the same base PQ and between the same parallel lines PK and QL.

$\therefore$ Area (ΔPOQ) $= \frac{1}{2}$ Area (parallelogram PQLK) ... (3)

Similarly, for ΔROS and parallelogram KLRS,

Area (ΔROS) $= \frac{1}{2}$ Area (parallelogram KLRS) ... (4)

Adding equations (3) and (4), we obtain

Area (ΔPOQ) + Area (ΔROS) $= \frac{1}{2}$ Area (parallelogram PQLK) +

$\frac{1}{2}$ Area (parallelogram KLRS)

Area (ΔPOQ) + Area (ΔROS) $= \frac{1}{2}$ Area (PQRS) ......(5)

Let us draw a line segment MN, passing through point OP and parallel to line segment PS.

In parallelogram PQRS,

MN || PS (By construction) ... (6)

PQRS is a parallelogram.

$\therefore$ PQ || RS (Opposite sides of a parallelogram)

$\Rightarrow$ PN || SN ... (7)

From equations (6) and (7), we obtain

MN || PS and PN || SN

Therefore, quadrilateral PNMS is a parallelogram.

It can be observed that ΔPOS and parallelogram PNMS are lying on the same base PS and between the same parallel lines PS and MN.

$\therefore$ Area (ΔSOP) $= \frac{1}{2}$ Area (PNMS) ... (8)

Similarly, for ΔQOR and parallelogram MNQR,

Area (ΔQOR) $= \frac{1}{2}$ Area (MNQR) ... (9)

Adding equations (8) and (9), we obtain

Area (ΔSOP) + Area (ΔQOR) $= \frac{1}{2}$ Area (PNMS) + $\frac{1}{2}$ Area (MNQR)

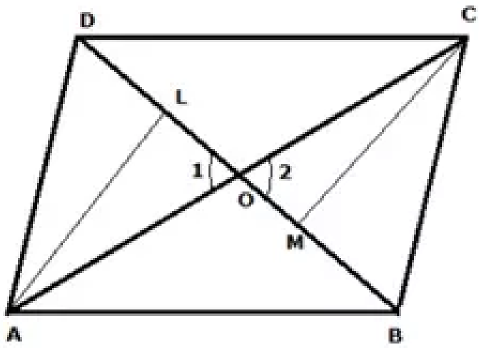
Area (Δ SOP) + Area (Δ QOR) = $\frac{1}{2}$ Area (PQRS) ....(10)

On comparing equations (5) and (10), we obtain

Area (Δ POQ) + Area (Δ ROS) = Area (Δ SOP) + Area (Δ QOR) =

$\frac{1}{2}$ Area($\parallel$ gm PQRS)

Answer 23.



Join AC. Suppose AC and BD intersect at O. Draw AL and CM perpendicular to BD.

ar(Δ ABD) = ar(Δ BDC)

Thus Δ ABD and Δ ABC are on the same base AB and have equal area.

Therefore, their corresponding altitudes are equal i.e. AL = CM.

Now, in Δ ALO and Δ CMO,

$\angle 1 = \angle 2$ (vertically opposite angles)

$\angle ALO = \angle CMO$ (right angles)

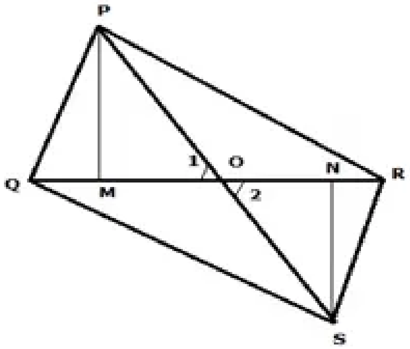
AL = CM

Therefore, Δ ALO $\cong$ Δ CMO (AAS axiom)

$\Rightarrow$ AO = OC

$\Rightarrow$ BD bisects AC

Answer 26.



Join PS. Suppose PS and QR intersect at O. Draw PM and SN perpendicular to QR.

ar(ΔPQR) = ar(ΔSQR)

Thus ΔPQR and ΔSQR are on the same base QR and have equal area.

Therefore, their corresponding altitudes are equal i.e. PM = SN.

Now, in ΔPMO and ΔSNO,

$\angle 1 = \angle 2$ (vertically opposite angles)

$\angle PMO = \angle SNO$ (right angles)

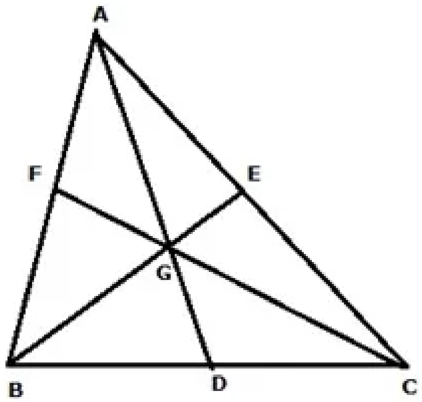
PM = SN

Therefore, $\Delta PMO \cong \Delta SNO$ (AAS axiom)

$\Rightarrow$ PO = OS

$\Rightarrow$ QR bisects PS

Answer 27.



The median of a triangle divides it into two triangles of equal areas.

In ΔABC, AD is the median

$\Rightarrow ar(\Delta ABD) = ar(\Delta ACD)$ ....(i)

In ΔGBC, GD is the median

$\Rightarrow ar(\Delta GBD) = ar(\Delta GCD)$ ....(ii)

Subtracting (ii) from (i),

$ar(\Delta ABD) - ar(\Delta GBD) = ar(\Delta ACD) - ar(\Delta GCD)$

$\Rightarrow \quad ar(\Delta AGB) = ar(\Delta AGC)$ ....(iii)

Subtracting (ii) from (i),

$ar(\Delta ABD) - ar(\Delta GBD) = ar(\Delta ACD) - ar(\Delta GCD)$

$\Rightarrow \quad ar(\Delta AGB) = ar(\Delta AGC)$ ....(iii)

Similarly, $ar(\Delta AGB) = ar(\Delta BGC)$ ....(iv)

From (iii) and (iv),

$ar(\Delta AGB) = ar(\Delta BGC) = ar(\Delta AGC)$ ....(v)

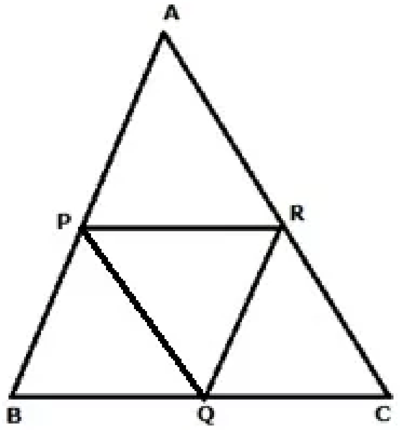
But $ar(\Delta AGB) + ar(\Delta BGC) + ar(\Delta AGC) = ar(\Delta ABC)$

Therefore, $3\, ar(\Delta AGB) = ar(\Delta ABC)$

$\Rightarrow ar(\Delta AGB) = \frac{1}{3}\, ar(\Delta ABC)$

Hence, $ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$.

Answer 28.



Since P and R are mid-points of AB and AC respectively.

Therefore, PR||BC and $PR = \frac{1}{2}BC$ ....(i)

Also Q is mid-point of BC,

$\Rightarrow QC = \frac{1}{2}BC$ ....(ii)

From (i) and (ii)

PR||BC and PR = QC

$\Rightarrow$ PR||QC and PR = QC ....(iii)

Similarly Q and R are mid-points of BC and AC respectively
Therefore, QR||BP and QR = BP ....(iv)
Hence, BQRP is a parallelogram.

$\Rightarrow$ PQ is a diagonal of ||gm BQRP

ar(ΔPQR) = ar(ΔBQP) ....(v) (diagonal of a ||gm divides it into two triangles of equal areas)

Similarly QCRP and QRAP are ||gm and
ar(ΔPQR) = ar(ΔQCR) = ar(ΔAPR) ....(vi)

From (v) and (vi)
ar(ΔPQR) = ar(ΔBQP) = ar(ΔQCR) = ar(ΔAPR)

Now, ar(ΔABC) = ar(ΔPQR) + ar(ΔBQP) + ar(ΔQCR) + ar(ΔAPR)
$\Rightarrow$ ar(ΔABC) = ar(ΔPQR) + ar(ΔPQR) + ar(ΔPQR) + ar(ΔPQR)
$\Rightarrow$ ar(ΔABC) = 4ar(ΔPQR)

$\Rightarrow$ ar(ΔPQR) = $\frac{1}{4}$ ar(ΔABC) ....(vii)

ar(||gm BQRP) = ar(ΔPQR) + ar(ΔBQP)

$\Rightarrow$ ar(||gm BQRP) = ar(ΔPQR) + ar(ΔPQR) (from (v))

$\Rightarrow$ ar(||gm BQRP) = 2ar(ΔPQR)

$\Rightarrow$ ar(||gm BQRP) = 2 x $\frac{1}{4}$ ar(ΔABC) (from (vii))

$\Rightarrow$ ar(||gm BQRP) = $\frac{1}{2}$ ar(ΔABC)

Answer 31.

Area (ΔPQR) = area (ΔPQS) + area (ΔPSR)....(i)

Since PS is the median of ΔPQR and median divides a triangle into two triangles of equal area.

Therefore, area (ΔPQS) = area (ΔPSR) .....(ii)

Substituting in (i)

Area (ΔPQR) = area (ΔPSR) + area (ΔPSR)

Area (ΔPQR) = 2area (ΔPSR) ....(iii)

Area (ΔPSR) = area (ΔPST) + area (ΔPTR) .....(iv)

Since PT is the median of ΔPSR and median divides a triangle into two triangles of equal area.

Therefore, area (ΔPST) = area (ΔPTR) .....(v)

Substituting in (iv)

Area (ΔPSR) = 2area (ΔPTR) ....(vi)

Substituting in (iii)

Area (ΔPQR) = 2 x 2area (ΔPTR)

Area (ΔPQR) = 4area (ΔPTR) ....(vii)

Area (ΔPTR) = area (ΔPMR) + area (ΔMTR) .....(viii)

Since MR is the median of ΔPTR and median divides a triangle into two triangles of equal area.

Therefore, area (ΔPMR) = area (ΔMTR) .....(ix)

Substituting in (viii)

Area (ΔPTR) = 2area (ΔPMR) ....(x)

Substituting in (vii)

Area (ΔPQR) = 4 x 2area (ΔPMR)

Area (ΔPQR) = 8 x area (ΔPMR)

area (ΔPMR) = $\frac{1}{8}$ area (ΔPQR)

Answer 32.

Since the diagonals of a parallelogram divide it into four triangles of equal area

Therefore, area of ΔAOD = area ΔBOC = area ΔABO = area ΔCDO.

$\Rightarrow$ area $\Delta BOC = \frac{1}{4}$area (||gm ABCD) ....(i)

In ||gm ABCD, BD is the diagonal

Therefore, area (ΔABD) = area (ΔBCD)

$\Rightarrow$area (ΔBCD) = $\frac{1}{2}$area (||gm ABCD)......(ii)

In ||gm BPCD, BC is the diagonal

Therefore, area (ΔBCD) = area (ΔBPC) ......(iii)

From (iii) and (ii)

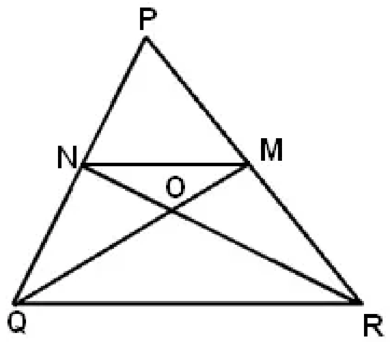
area (ΔBPC) = $\frac{1}{2}$area (||gm ABCD) ....(iv)

adding (i) and (iv)

area (ΔBPC) + area ΔBOC = $\frac{1}{2}$area (||gm ABCD) + $\frac{1}{4}$area (||gm ABCD)

Area of OBPC = $\frac{3}{4}$ area of ABCD

Answer 33.



Join MN. Since the line segment joining the mid-points of two sides of a triangle is parallel to the third side; so, MN||QR

Clearly, ΔQMN and ΔRNM are on the same base MN and between the same parallel lines.

Therefore, area (ΔQMN) = area (ΔRNM)

⇒Area (ΔQMN) – area (ΔONM) = area (ΔRNM) – area (ΔONM)

⇒area (ΔQON) = area (ΔROM) ....(i)

We know that a median of a triangle divides it into two triangles of equal areas.

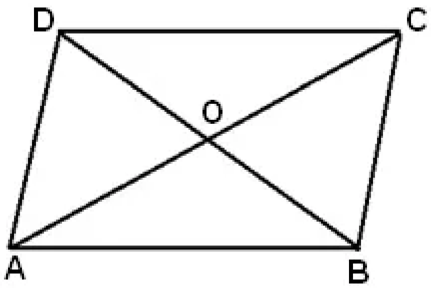
Therefore, area (ΔQMR) = area (ΔPQM)

⇒area (ΔROQ) + area(ΔROM) = area (quad. PMON) + area (ΔQON)

⇒area (ΔROQ) + area(ΔROM) = area (quad. PMON) + area (ΔROM) (from (i))

⇒area (ΔROQ) = area (quad. PMON)

Answer 37.



Since the diagonals of a parallelogram bisect each other at the point of intersection.

Therefore, OB = OD and OA = OC

In ΔABC, OB is the median and median divides triangle into two triangles of equal areas

Therefore, area (ΔBOC) = area (ΔABO) ....(i)

In ΔADC, OD is the median and median divides triangle into two triangles of equal areas

Therefore, area (ΔAOD) = area (ΔCDO) ....(ii)

Adding (i) and (ii)

area (ΔAOD) + area (ΔBOC) = area (ΔABO) + area (ΔCDO)